# The Appearance of the Scitovsky Paradox in the Coase Theorem

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**AIMS OF THE PAPER**
Our study aims to interpret the Coase theorem in a way which also allows us to interpret the basic doctrine within the logical space of the Kaldor-Hicks-Scitovsky tests. In doing this we try to prove that, in the case of stakeholders with conflicting interests in using the environment (the so-called "environmental conflict") the relevance of the Coase theorem may be questioned by the Welfare test recommended by Scitovsky. Our further aim is to define those limiting conditions under which the conflict amongst stakeholders may still be able to produce Pareto's efficient allocation of resources.

**METHODOLOGY**
Our research is based on the recognition that Coase's recommendation to compensate the welfare losses generated by external effects, can be checked by the Kaldor, Hicks and s Scitovsky tests. After showing the isomorphism latent in both concepts, we concentrate on how the options of compensation between actors A and B are formulated if the law regulating the use of the common environment (compared to the earlier version) shows the person who must bear the burden as different from the one who has the benefit. In respect of our research aims, we considered comparative statistical analysis as being the most appropriate.

**MOST IMPORTANT RESULTS**
We have explored further conditions or limitations relating to the socially tolerable or desirable value of the externalities (both burdens and benefits), and, in general theoretical terms, the optimal distribution of resources. Our results ultimately support those views which give rise to (at least some) doubts regarding the general interpretability of Pareto's efficient allocation of production factors, or the Pareto Improvement. Our research has also shed light on the fact that, under an *extreme* ratio of resource distribution (in respect of the division of externalities close to 50:50), the chances of achieving the Pareto optimal improve, whilst otherwise they worsen.

**RECOMMENDATIONS**
Our results suggest that we need to have some reservations in respect of the theory and practice of the "market-conform" treatment of external economic impacts. We further conclude that, in the economic space burdened with external impacts, more consideration should be given to structural regulation which is more tolerant of the concentrated spread of inputs, in case we should face an "all or nothing" situation in terns of the comfortable use of the environment.

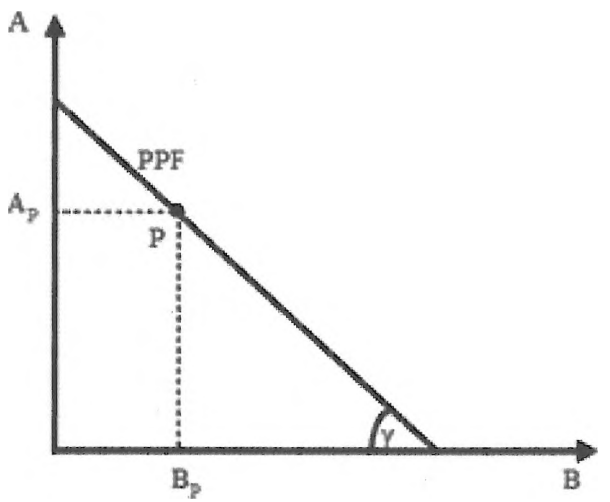
*Keywords: Coase Theorem, Kaldor-Hicks-Scitovsky tests, environmental comfort/discomfort, externality*

## PREAMBLE

The current spread of environmental pollution theories in economics would seem to suggest signs of a happy shift of focus. This statement, however, certainly does not refer to the appalling devastation of our planet, but rather to the increase in the opriority of professional literature aiming at understanding and fighting back this phenomenon. From among the concepts formulating also economic policy recommendations reflecting on the mentioned topic the "conventional" *Pigou* inspired conception (Pigou 1912; 1932) and *Coase's* market conform view (Coase 1959; 1960; 1988) may be highlighted. The conflict between the two approach fluctuates: the practice of the European Union – e.g. in the case of ISO 14001 and EMAS III standards – explicitly points towards the self-regulatory mechanisms generated by legal-market frameworks, however, parallel to this, as the member of the Pigou Club founded in 2006 by Mankiw, renowned economists (e.g. Krugman, Nordhaus, Becker) do not let the virtues of the rival approach to be forgotten either.

With our study we would like to complete the profession's "compulsory" provisos and corrections related to the Coase theories – which are summarised at a high level for instance by Cullis & Jones (2003, 53-57), Kerekes – Szlávik (1999, 111-112), Szücs (2012), and Bartus – Szalai (2014). In the course of this, we will aim at interpreting the Coase theorem in a way that allows us to explain this doctrine within the logical space of the Kaldor-Hicks-Scitovsky tests (Kaldor 1939, Hicks 1940, Scitovsky 1941). After having made visible the isomorphism hidden in the two system of thoughts, we will attempt to prove that in the case of *certain types* of the economic effects called by Marshall and Pigou as "external" the Coase theorem can fall the control recommended by Scitovsky. There are many ways to interpret or explain the basic idea of Coase (Bartus – Szalai 2014, Medema & Zerbe 2000). Generally from among this each deals with the fact that if the rights related to the use of the environment are clearly declared and the costs of bargaining by the parties involved in an economic interaction are insignificant then the Pareto efficient

*Figure 1. Production possibility frontier in the case of constant returns*
*Source: own design*



allocation of the resources, as well as that of the effects entailed through their use will take place even without official intervention.

## MODEL ASSUMPTIONS, THE USED METHODOLOGICAL AND CONCEPTUAL SYSTEM

We will illustrate our thoughts within a scope of a very simplified model using only two actors. We assume that our actors, *A* and *B* produce the goods – marked with the same letters, i.e. A and B – utilizable/ used by them by sharing the same amount of resources *(F)*. If α and β variables are the quantities used from *F* resource, is true that

$$F = \alpha + \beta \qquad (1)$$

The range of output combinations pertaining to the total possible distribution of resources are represented on Figure 1 by the points on the (transformation) curve and illustrate the *production possibility frontier (PPF)*. This shows the *constant returns* of actors (assumed only for the sake of simplicity), i.e. a constant *(tg γ)* transformation rate. The coordinates of any point P on the curve ($A_p$ and $B_p$) refer to the composition of the output combination that is realised in the case of a concrete allocation of *F*.

We should now introduce the concept of *environmental comfort/discomfort* [from now referred to as (dis)comfort]! By this we mean conditions, circumstances, phenomena, manifestations which are *equally* visible through *A* and *B*, although they exert a contrary influence on their effectiveness in the creation/existence or termination/ absence of which according to the sense parties' *disincentive* – so called "*environmental conflict*" (Szántó 2008) – subsists. In fact, the previous concept covers an "externality" emitted (or maintained) by either of the parties and *suffered* by the other (hence, in our study an exclusively *negative* externality). However, we deliberately avoided the use of the term "external economic effect" in the definition. One explanation for this is that the respective effect internalises in the currently examined models. On the other hand, we formulate in a natural manner to highlight the *reciprocity, mutuality* of the "externality" viewed by Coase and not the *unidirectional* Pigou "verticality" of the conventional "emitter-offended" roles in order to avoid evoking the conceptual reflexes attached to this latter. As Coase formulates: "The conventional approach [linked to Pigou] had put the nature of the decision to make in the shade. In general, the question arises in the form that *A* causes damage to *B* and what has to be decided is how we should hinder *A from* doing this. This however is incorrect. We are dealing with a mutual issue. In order to hinder *B's* damage we actually cause damage to *A*. The real question to decide is whether or not *A* should be able to cause damage to *B*, or *B* should be able to cause damage to *A*. The task is to avoid the greater damage." (Coase 1960, 140, *B.J.'s supplement.*)

Since manifestations of the environmental (dis)comfort modulate the efficiency of the actor's activity, these also influence the position of the *PPF*-curve. This effect – depending on the type and strength of the (dis)comfort – may manifest itself in different forms and with different "vehemence". In the following we will analyse only the two basic types whilst certainly not excluding the existence and relevance of the others. Our examination first focuses on mapping the *inversion, "sign changes"* of the phenomenon between *A* and *B*, then we will seek to find an answer as to the chances of the effectiveness of the Coase theorem in each case. By the inversion of (dis)comfort we mean the exchange of the preferences of environmental right – and, through this, that of the beneficiaries of the same (by holding as an example in the stereotype case of smokers – non-smokers the consequences of a conversion of the right to clean air and the right to smoke).

## KEY TYPES OF ENVIRONMENTAL (DIS)COMFORT

In the course of our analysis we assume in general and arbitrarily that the *initial* conditions perceivable by both *A* and *B* are considered more beneficial to the *latter* (which in conventional terms may mean that *B* can pursue their activity that is disturbing for *A* freely without any restrictions; but it may also mean that *A* makes sacrifices in order to stifle the natural, however in *B's* perspective unpleasant, effects of their respective operation). What will happen to the *PPF*-curve if due to any reasons the sign of (dis)comfort becomes opposite (i.e. *B* eliminates, neutralises their own environmental emission, or takes on that of *A*)? When answering, we imagine the inversion for the time being as a *jump* (discrete and non-gradual transition) between two points of a scale expressing the (relative) strength of the "extern effect" in a meaning that we define it as the appearance/disappearance of an effect of *fixed* extent (or rate).

In the *first* examined case the (dis) comfort influences the *general* – vice versa unfavourable/beneficial – environmental condition of the activity of *A* and *B*. The expression "home field advantage" might be the best to characterise its essence. Its effect manifests itself in the fact that it increases the output of one actor by a constant %, while decreasing the output of the other actor by a constant % compared to the original (opposite) situation – similar to the athletes' performance shown in home versus foreign environment. The shape of production functions valid here may be for instance

$$A = \alpha k^{aE} \tag{2}$$

and

$$B = \beta k^{-bE} \tag{2a}$$

where $k \geq 1$ is the constant referring to the *"basic intensity"* of the „externality", the interpretation of the other markings is unchanged. Further, *a* and *b* are some non-negative "technological" constants, and *E* as a variable mark the "dose of externality". In the following we assume that for our actors the perfectly free success or perfect blockage of the *spill over* effect mean the *positive* and *negative* versions of the *very same* number *(E)* – certainly *vice versa*.

*Coase's* view – as seen – establishes the opposing interest of *A* and *B* regarding the method of use of the environment, and the right is far from absolute, far from eternal – so to say *ad hoc* – they leave the allocation of the *emitter and offended* roles to their attitude. Therefore, the values of *E* for certain actors are *symmetric* compared to the origin. The opposite signs of *E* refer to the effectiveness thus consequences of an environmental effect of a specified extent that are opposing *A* versus *B*.

According to Figure 2 the appearance of (dis)comfort results in the transfer of each of the points of the *PPF*-curve that is, the *new curve is generated as the mapping of the old one.* (Hereinafter the curve characteristic of the use of environment beneficial for *B* is referred to as $PPF_B$ while in the case comfortable to *A* by $PPF_A$.) The mutually corresponding points *($B_{0\to1}$, $A_{0\to1}$, $N\to P\to M$)* symbolise the output combinations prevailing at the *same allocations rates of the F* resource in the case of direction change of the *spillover*. We drew our figure deliberately *symmetrical* by assuming $a = b$, on one hand, in order to express the mentality of the Coase approach and, on the other hand, to display certain correlations to be covered soon in a more plastic – albeit simpler – manner.

As seen, the exchange of such (dis)comfort triggers in *A's* production $A_1/A_0$-times efficiency improvement, and in *B's* production $B_1/B_0$ efficiency decrease, where

$$A_1/A_0 = \frac{\alpha k^{aE}}{\alpha k^{-aE}} = k^{2aE} \tag{3}$$

and

$$B_1/B_0 = \frac{\beta k^{-bE}}{\beta k^{bE}} = k^{-2bE} \tag{3a}$$

*Figure 2. Shift of the PPF-curve resulting from the inversion/transfer of the "home field advantage" type of right to use the environment*
*Source: own design*

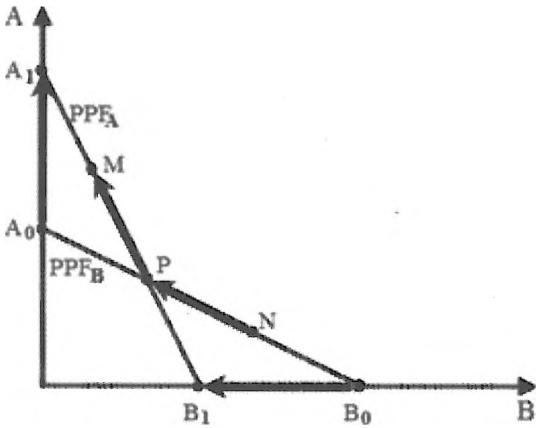
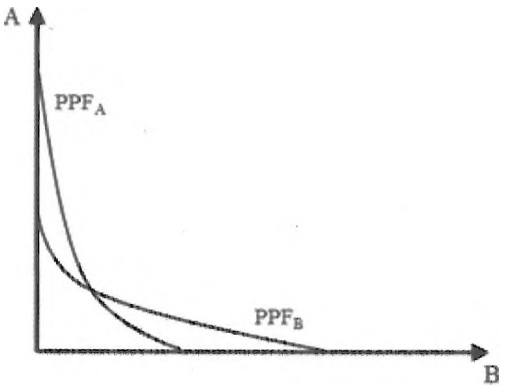



*Figure 3. Inversion of (dis)comfort depending on the actors' scale of activity*
*Source: own design*




In the above formulas the strength of the effect related to the use of the environment depends – as seen – basically on two – $k$ and $E$ – factors, and accessorily on the values of $a$ and $b$.

When presenting the *second* version we assume that the strength of (dis)comfort can be linked (also) to the *rate of sharing of the resource* by the two partners. When drawing Figure 3 – pursuing to this – we started from the fact that the efficiency of the operation of the actors (over and above the preferences regarding the right to use the environment) changes in the same direction as the number of factors used by their own, and in the opposite direction compared to the number used by the other party. This can be seen if the "(de)formity" of the operating environment also depends heavily on the (relative) *scale* of the actors' activity, or the emission.

Here the possible formulae of production functions are

$$A = \alpha k^{\frac{\alpha a^E - \beta b^{-E}}{F}} \tag{4}$$

and

$$B = \beta k^{\frac{\beta b^{-E} - \alpha a^E}{F}} \tag{4a}$$

where $a$ and $b$ indicate non-negative technological parameters characteristic of $A$ activity, while â and  indicate the same for $B$ activity, although the different levels of $E$ can be interpreted this time quite explicitly as the "dose of environmental right possessed by the individual parties".

When analysing function (4) we can see that the increasingly large *positive* values of $E$ (which, regarding liability, refer to *A's high exemption, privilege*) *ceteris paribus* assign an increasingly larger exponent to $k$, compared to the case when a *negative* sign indicates the opposite direction of the right. This means that the dependence of *A's* efficiency on *B's* activity decreases proportionately to the increase of *B*'s obligation by regulation to cut their emission. The *zero* level of $E$ reflects - in this case – a sort of an *"exlex"* condition, when the strength of "external" effects mutually perceived by producers depends directly on the distribution rate of resources, is *non-modulated* by the law and will be neutralised only in the case of 50-50 % allocation of the amount of $F$ factor. The perfectly *independent - from the beginning externality-free* - activity of the two actors would be symbolised by $k = 1$.

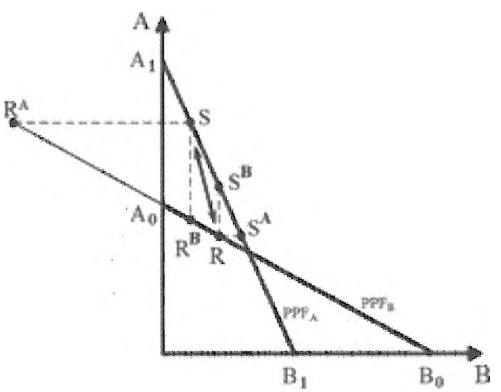
Further, it is true that, the high values of α and the accordingly low values of β would again result in the increase of the exponent, i.e. the environmental effect negative for *A* and, triggered by *B's* production, will mitigate by the reduce the factors used by the unpleasant partner.

## THE USE OF KALDOR-HICKS AND SCITOVSKY TESTS FOR THE COASE-TYPE MANAGEMENT OF EXTERNALITIES

We will carry out examinations related to the Kaldor-Hicks and Scitovsky tests regarding the *"first type"* (dis)comfort that is easier to manage in mathematical terms. Since – as seen – the *PPF*-curves of the first and second case are similar (if not in all, but in many respects), by analysing the simpler version we can produce statements which are valid also for the more complex relations. Let us look at Figure 4 – shown as the corresponding interpretation of Figure 2, according to which a mutual mapping exists between the $S$ and $R$ points of two *intersecting PPF*-curves.

The validity of $S$ means for $A$, while the validity of $R$ means for $B$, a favourable legal regulation regarding the environment. Let us assume that $B$ actor attempts to "purchase" from $A$ the right to the use of the environment, which on our figure would correspond to an $S \rightarrow R$ shift. According to the Kaldor-Hicks test, this action is desirable from society's perspective, meaning that a *potential Pareto improvement* is pos-

*Figure 4. Kaldor- and Scitovsky-tests for the attempt to obtain the environmental right*
*Source: own design*



sible, if the purchaser is able to *successfully* compensate the original beneficiary of the right. In our model, *A's* compensation may be shown as the shift of $A_0$-$B_0$ including point *R* on the *PPF*-curve, and can be interpreted as the regrouping (re-allocation) of a part of the *F* resource between $B \rightarrow A$.

*The compensation is successful, if one actor's willingness to compensate is not smaller than the other party's demand for compensation.* The *B* actor's *willingness* to compensate is the greatest sacrifice with the assuming of which they would still just not be in a worse situation than it was originally in before purchasing the environmental right. This is indicated by point $R^B$, which reflects the possibility to create *B* quantity included in *S*. *A's demand* for compensation – the still acceptable limit of which would be illustrated by point $R^A$ falling already outside the coordinate system – is likewise attached to ensuring the output level before the transfer of the environmental right (corresponding also to *S*). Since this – as we can see – exceeds the *B* actor's willingness to compensate, the action to obtain the right will be *unsuccessful.* All this refers to the fact that, for society, *A's free* use of the environment would be desirable. This will, however, only be certain in case the result of the Kaldor-Hicks test is proven also by Scitovsky's counter test. A counter test would now mean starting from the fact that the environmental right is originally possessed by *B* was acquired by *A*. Thus, roles are switched and this time it is *A's* task to compensate. The willingness to compensate would be limited – *mutatis mutandis* – by point $S^A$, while *B's* minimum demand for compensation would be analogous to reaching point $S^B$. In our case the action is *successful,* if the purchase of the environmental right enables a potential Pareto improvement. *A successful Scitovsky test is at the same time equivalent to the validity of the Coase theorem, i.e. independent from the original legal regulation the bargaining of the economic actors deliver the same final result regarding the method of the use of the environment*

## APPEARANCE OF THE SCITOVSKY PARADOX IN THE LOGICAL SPACE OF THE COASE THEOREM

Let us now go back to Figure 2! Firstly, let us establish that, due to their symmetrical layout, the length of $A_0$-$B_0$ and $A_1$-$B_1$ curves is identical, and the distance of their mutually mapped points from *A* or *B* axial section is the same. Following this, let us realise that, as a result, point *P* of $A_0$-$B_0$ curve corresponds to point *M* of the $A_1$-$B_1$ curve, although, the length of $A_0$-*P* and $A_1$-*M* sections is the same. Due to the same consideration, it is also true that the mapping of point *N* of $A_0$-$B_0$ curve is point *P*, and the length of section $B_0$-*N* and section $B_1$-*P* is the same. Curve segments *M*-*P* and *P*-*N* - displayed by a bold line on the figure - include such output combinations corresponding to one another in the case of sign change of the (dis)comfort, the existence of which undermines the validity of Coase's theorem related to the *clarity* of optimal allocation. In this range of the mutual mappings between the two curves namely the Scitovsky counter test fails and, through this, Coase's doctrine is not met.

As an illustration of this we will examine on Figure 5 what would happen if the initial output combination is represented by a certain point, point *G* of section *P*-*N*. It can be acknowledged that *A's* attempt to obtain the environmental right is unsuccessful, as from point *H,* available through this, it could satisfy *B's* demand for compensation only in a "self-destructive" manner. The Kaldor-Hicks test would thus consider *B's* environmental comfort and *A's* discomfort as beneficial for the society. However, the Scitovsky counter test leads to a paradox: if originally *A* disposed over the right to freely use the environment and we would set out from *H* combination, after reaching point *G, B* would be unable to satisfy the demand for compensation, which would suggest that in terms of the society not *B's* but *A*'s disposition over the environment would be beneficial, thus contradicting the previous conclusion.

What depends the emergence of the previously described contradictory situation on? First of all we must make clear that it depends on the *type* of the (dis)comfort under discussion, the change in sign of which can be illustrated as the shift of the *PPF*-curve. We saw that the paradox assumes *intersecting* curves, which is typical following the inversion in the case of most types of the "externalities".

The following factor is represented by the *sharing ratio* of the resource allocated between the parties. From this depends namely whether or not the mutually corresponding points of the two curves are located on section *M-P-N* or outside it. When looking at Figure 2 it appears that the *chances of impossibility of the Coase doctrine are smaller if the distribution rates are extreme.*

Further, the likelihood of the emergence of Scitovsky paradox is closely related to the *fraction* of the whole curve covered by the implicated sections of the *PPF*-curves. For the symmetrical case examined by us we can see that, based on Figure 6, this ratio is nothing else (when the intersection of the curves is located on a 45° bisector) but

$$\theta = (f - g)/(f + g) \qquad (5)$$

and its value is *zero* if $f = g$. By taking formulae (2) and (2a) as a basis, this would be met in the case where $k = 1$, and/or $E = 0$, when $\lambda = \mu = 45°$ means that the inversion of the (dis)comfort – due to the insignificant effect of the "externality" on the production – does not influence the position of the *PPF*-curves (or output combinations) on the merits, and thus they cover each other. The origin of the intensity and 'dose' of externality can be best characterised (provided that $a = b = 1$) by

$$\omega = \frac{A_2}{A_0} = \frac{B_0}{B_2} = k^{2E} \qquad (3b)$$

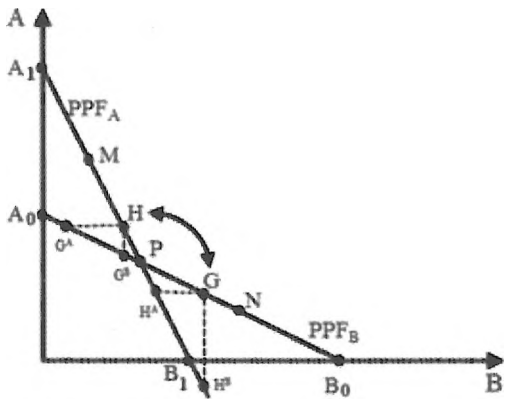
ratios which measure the *strength* of the change in efficiency resulting from the inversion.

We can easily acknowledge that

$$\frac{\omega - 1}{\omega + 1} = \theta \qquad (5a)$$

i.e. with the increasing strength *() of* an *undividable* "externality" – which may be the result of the increased values of $k$, and/or $E$, maybe $a$, $b$ – the proportion *()* of the set of factor combinations affected by the Scitovsky paradox heads to *one. Thus, the stronger the role of the (dis)comfort in influencing the efficiency, the greater the likelihood that the Coase theorem will not be met,* the "optimal condition" of the

*Figure 5. Collision of the Scitovsky paradox and Coase theorem*
*Source: own design*



environment and the desirable allocation of resources cannot be determined.

In the above we assumed that environmental (dis)comfort is an *(in)divisible* dimension, i.e. due to the lack of continuity of the inversion there is no constant transition between the position of the two *PPF*-curves. In the following we attempt to prove that the *continuity* of the dose of "externality" *(E)* can reduce the risk of emergence of the Scitovsky paradox. We note that in this case $E = 0$ has a double meaning: it either means the perfect lack of (dis)comfort or suggests a *compromise* representing the natural, 50-50% "allocation" (and enforcement) of environmental rights between *A* and *B*.

## TRAJECTORIES OF THE OUTPUT COMBINATIONS IN THE CASE OF CONTINUITY OF THE INVERSION OF (DIS)COMFORT

Again we start from the fact that the environmental conditions are originally comfortable for *B*, where curve $A_0$-$B_0$ can be considered valid. This time our task is to determine and explore the trajectories that constitute the "route" of the individual points of the *PPF*-curve towards $A_1$-$B_1$ in the plane of the output combinations in the case

*Figure 6. The ratio of output combinations affected by the Scitovsky paradox*
*Source: own design*

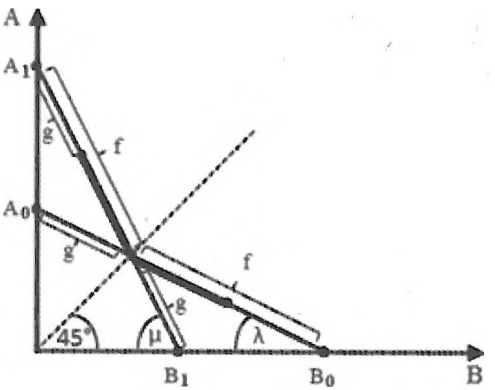
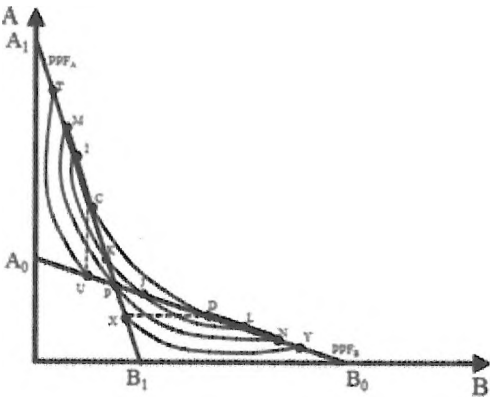



*Figure 7. Trajectories of output combinations in the course of continuous "change in sign" of the environmental right*
*Source: own design*



of *continuous* modification of the value of *E*. The significance of the information so obtained is that certain ranges of the paths mentioned may symbolise Pareto efficient situations compared to the points of the initial and final *PPF*-curves, the existence of which would improve the possibility to obtain compensation due when acquiring the environmental right. On Figure 7, derived through supplementing Figure 2, we can follow the results of our examination. (For the sake of better view the angle of inclination of the *PPF*-curves are somewhat distorted compared to the original display.)

What is interesting for us is that in the range jeopardised by the Scitovsky paradox the *divisibility* of (dis)comfort can somewhat restore the relevance of the Coase theorem through the emergence of the *C-D* (best) trajectory. This means that the Coase doctrine will again be valid for the attempts to purchase the environmental right – setting out from the points of sections *C-P* and *P-D* of the *PPF*-curves –, as the "offering" of the *C-D* path enables a Pareto improvement, i.e., successful compensation. The fact that reaching the *C-D* arch corresponds only to a *partial* re-allocation of the (dis)comfort, however, suggests an important difference compared to the original assumptions of our model, as the total change between the two extreme values of *E* does not occur. Let us consider the following: tests setting out from the points of sections *M-C* and *D-N* continue to remain within the "scope" of the Scitovsky paradox. (It might be promising to apply the possibility of divisibility to the "original" context of the Kaldor–Hicks- and Scitovsky tests).

The attempts to transfer the right setting out from the points of *U-P* and *X-P* sections cast light on additional interesting possibilities of the bargaining described by the Coase theorem. It is namely clear that this time *two types* of solution are offered to the parties. On the one hand they can opt for the exchange of the *entire* liability, resulting in a Pareto improvement, since reaching *T-M* or *Y-N* sections from there, they can *successfully* compensate their partner. On the other hand, they can enter into negotiations regarding the *partial* transfer of the right, since, from the points of *U-P* and *X-P* sections "sagacity opens" for them on the optimal trajectories.

The motives that in such cases determine the selection of one or the other option of the bargaining process may be subject to examinations that point beyond our study. We note here that even correlations, conclusions that can be formulated in the abstract world of our model can be overwritten by the high degree of freedom of the features typical of *concrete* market circumstances, e.g. more than two actors not uniformly affected by the external effect, as well as more than one resource not necessarily subject to rivalry by the actors or the stability of market prices, etc. (See, for example Heindl's high-level critical remarks: 2009, 2.1.)

It is remarkable that points *C* and *D* are *bisectors of* $A_1$-$B_1$ and $A_0$-$B_0$ sections, meaning that the trajectory serving as the Pareto optimum – *by reflecting the features of our model* – generates if *F* resource quantity is distributed 50:50. We will prove this correlation by means of a derivation but within a *more general* scope.

Let us start from the generalised forms of (2) and (2a) production functions:

$$A = \alpha^x m^{\alpha E} \qquad (2^*)$$

and

$$B = \beta^y n^{-bE} \qquad (2^*a)$$

where the values of *x* and *y* non-negative constants as the exponents of α and β relative to 1 refer to the *increasing, decreasing or constant* nature of the return that shows in the production of *A* and *B*. Further, by using *m* and *n* parameters instead of the uniform *k*, we allow also the deviation of the (dis)comfort intensity at the two allocation areas. Our derivation intends to show the value of β that maximises *A's* output

by fixing $E$ at a discretional level. We can easily acknowledge that $\alpha$ – *ceteris paribus* – linked to the thus obtained $\beta$ pursuant to (1) likewise maximises *B's* value, i.e. we arrive at the resource allocation that delivers the Pareto efficient trajectory.

Let us express $E$ from (2*a):

$$E = \frac{\lg \frac{\beta^y}{B}}{b \cdot \lg m} \tag{6}$$

then substitute it in (2*) by also enforcing correlation (1). By settling the obtained formula

$$A = (F - \beta)^x \left(\beta^y / B\right)^{\frac{a \ln m}{b \ln n}} \tag{7}$$

then from this we arrive at

$$A = \frac{(F-\beta)^x \beta^{\frac{ya \ln m}{b \ln n}}}{B^{\frac{a \ln m}{b \ln n}}} \tag{7a}$$

forms. We search for the maximum of $A$ with derivation:

$$\frac{dA}{d\beta} = \frac{1}{B^{\frac{a \ln m}{b \ln n}}} \left[ \frac{\frac{ya \ln m}{b \ln n}}{(F-\beta)^x \beta^{\frac{ya \ln m}{b \ln n} - \lambda}} - x(F-\beta)^{x-\lambda} \beta^{\frac{ya \ln m}{b \ln n}} \right] \tag{8}$$

At the derived zero place we arrive at the

$$\beta = \frac{Fay \cdot \ln m}{bx \cdot \ln n + ay \cdot \ln m} \tag{9}$$

Correlation, from which it becomes clear that at the *symmetry* of the $A$ and $B$ production functions enforced in our models that is, if $a = b$ and $x = y$, and $m = n$ (= $k$) the Pareto-efficient trajectory is linked to the 50-50% allocation of the resource. Then $\alpha = \beta = F/2$ irrespective of the absolute values of *a, b, x, y, and m and n.* We note that the version of the derivation *mutatis mutandis "focused" on A* delivers

$$\alpha = \frac{Fbx \cdot \ln n}{bx \cdot \ln n + ay \cdot \ln m} \tag{9a}$$

As a final result with "symmetrical" interpretation possibilities, we can see that the total of the formulae (9) and (9a) is – as expected – $F$.

A further interesting fact concerning our results is that the range of the Pareto efficient re-allocation of the resource (and the environmental right) is only influenced by the parameters of the production functions (as constants) irrespective of the output level of the individual goods. From another perspective this means that this range is made up of the best points of the *very same* trajectory and not that of several intersecting paths.

## SUMMARY

Our study undertook a review of Coase's well known theorem. According to the doctrine in question, if the rights related to the use of the environment are clearly defined and the costs of bargaining by parties involved in the economic interaction are insignificant, the (Pareto) efficient allocation of resources takes place also without regulatory intervention.

We explained our thoughts within the scope of the two-actor model which enabled the use of the Kaldor-Hicks, as well as the Scitovsky tests. We assumed that our actors, $A$ and $B$, can produce the goods – indicated with similar letters – used by them by sharing a certain amount of resource *(F).* We illustrated the set of output combinations pertaining to all possible distribution(s) of the resource with the points of the *PPF*-curve, the features of which referred to the nature, direction and strength of the "externality". Our examination first focused on the inversion, "change in sign" and mapping of the different types of environmental effects beneficial-adverse for $A$ versus $B$, tnd then we sought an answer as to the chances of meeting the Coase theorem in the individual cases.

We proved that, if the changes in the environmental comfort of the economy generate a new *PPF*-curve having an *intersection* with the original curve, the relevance of the Coase theorem can be challenged by the welfare test recommended by Scitovsky. We could see that the extent of "externality" tolerable by society and the Pareto efficient distribution of related burdens cannot be determined in this case - even theoretically.

The most important theoretical benefits of our model are:

- By introducing the system of *PPF*-curves, we created a new classification possibility of the types of environmental effects important in terms of the scope of validity of the Coase theorem.
- We proved that, with the stronger manifestation of certain types of environmental (dis)comfort, the determination of the "economically optimal" resource allocation (and environmental pollution) can become impossible.
- By distinguishing *divisible* and *non-divisible* externalities, we pointed out that the plausibility of the Coase theorem is increased by the continuity of the extent of "external effect", whilst it is decreased by its non-divisibility
- Our research shed light on the fact that, given the *extreme* distribution rate of the resource (nearly 50:50 in the case of the divisibility of the "externality") the chances of arriving at a Pareto optimal allocation *improve,* whilst, in the case of a maintained rate, the chances deteriorate.

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